Thermohydraulic Simulation on CIC Conductor with Adaptive Mesh Finite Volume Method for KSTAR Tokamak Superconducting Magnet

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Abstract —**To study the quench in the CICC, the numerical analysis code was developed. The fully implicit time integration of upwind scheme for finite volume method is utilized to discretize the equations on the staggered mesh. The scheme of adaptive mesh is proposed for the moving boundary problem and the time term is discretized by the θ-implicit scheme. The discretized equations are solved by the IMSL. The error analysis of this method is performed by various step-sizes of time and space. The thermal hydraulic behavior of the CICC used in KSTAR is studied.**

***Index Terms*— CICC, Staggered mesh, Finite volume method of upwind scheme, Fusion superconducting magnet.**

I. INTRODUCTION

Superconducting magnets have many applications in the industry and large-scale experimental devices. The Korean Superconducting Tokamak Advanced Research (KSTAR) project is a full superconducting Tokamak device with the central magnetic field of 3.5 T at the major plasma radius of 1.8 m. The Tokamak adopts a cable-in-conduit conductor (CICC) as the basic elements for the TF and PF superconducting magnets. The CICC, which is formed by cabling of superconducting strands and pure copper strands sealed in stainless steel or Incoloy conduit, is characterized by the large stability margin, high breakdown voltage, good mechanical strength and smaller total mass of liquid helium necessary for the cooling and lower AC losses. It has been widely employed in fabricating large-scale superconducting magnets. For the safety operation of the superconducting magnets, the protection during the quench of the system is one of the important issues. Therefore, predicting the quench characteristic, especially, the maximum supercritical helium pressure in the conduit, hotspot temperature of the superconducting strands is essential to issues for the designers. The code assumes that the temperature of supercritical helium and superconducting strands is the same due to the very high heat transfer phenomena between the supercritical helium and superconducting strands [1]. The governing equations are one-dimensional fluid dynamic equations for the supercritical helium and the equation of heat conduction for the conduit. A finite volume method of upwind scheme on the staggered mesh discretizes the partial derivative of space for the equations. The time integration is employed by the θ-implicit scheme. The subroutines in the IMSL are used to solve the linear sparse system. The adaptive mesh is presented to generate the grids movements and the numerical implementation is introduced in this paper.

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II. PHYSICS CHARACTERISTICS OF QUENCH IN CICC

The CICC contains superconducting strands, pure copper strands, supercritical helium and conduit. While the length of the conductor for the Tokamak magnets is the dimension of 10^2-10^3 m, typically, the transverse scale of the CICC is the order of 10^{-2} m. Thus, one-dimensional model is reasonable to assume for the thermal hydraulic analysis. The heat conduction of supercritical helium is neglected since the heat diffusion is much lower than that of convection. The temperature difference can be ignored and the temperature of superconducting strands and helium are the same [2]. However, the temperature difference between the helium and the conduit should be considered because of a small wetted perimeter of conduit. The temperature distribution of the conduit is predicted by the heat conduction equation. The current redistribution among superconducting strands is neglected in this model. The coupled equations for the supercritical helium, superconducting strands and conduit are expressed as:

$$M\frac{\partial\psi}{\partial t}+A\frac{\partial\psi}{\partial x}+B\psi=\frac{\partial}{\partial x}\left(K\frac{\partial\psi}{\partial x}\right)+G \qquad (1)$$

where the unknown augment matrix of ψ and coefficients matrix of the equations are defined as :

$$\psi=\begin{pmatrix}\rho\\ u\\ T_{he}\\ T_{jk}\end{pmatrix},\quad M=\begin{pmatrix}1&0&0&0\\0&\rho&0&0\\0&0&\rho C_t&0\\0&0&0&\gamma C_{jk}\end{pmatrix},K=\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&k_{st}&0\\0&0&0&k_{jk}\end{pmatrix},$$

$$A=\begin{pmatrix}u&\rho&0&0\\ \frac{1}{\rho\alpha}&\rho u&\frac{\beta}{\alpha}&0\\ 0&\left(\frac{A_{he}}{A_c}\right)\frac{\beta}{\alpha}T_{he}&\rho u C_v\left(\frac{A_{he}}{A_c}\right)&0\\ 0&0&0&0\end{pmatrix},G=\begin{pmatrix}0\\0\\q_{dst}+q_{jst}\\q_{djk}+q_{jjk}\end{pmatrix}$$

$$B=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & f\rho\frac{|u|}{2d_h} & 0 & 0 \\ 0 & -f\rho\frac{|u|u}{2d_h}\left(\frac{A_{he}}{A_c}\right) & \frac{p_{jk}h_{jk}}{A_c} & -\frac{p_{jk}h_{jk}}{A_c} \\ 0 & 0 & -\frac{p_{jk}h_{jk}}{A_{jk}} & \frac{p_{jk}h_{jk}}{A_{jk}} \end{pmatrix}$$

$$\rho C_t=\frac{A_{he}}{A_c}\rho C_v+\gamma C_{st}$$

In the equations, $\boldsymbol{x}$ and $\boldsymbol{t}$ indicate the coordinates of space and time, respectively. $\boldsymbol{\rho}$, $\boldsymbol{u}$ and $\boldsymbol{T_{he}}$ are, respectively, the density, velocity, and temperature of supercritical helium. $\boldsymbol{T_{jk}}$ is the conduit temperature. $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are the bulk compressibility and expansion coefficients of supercritical helium. $\boldsymbol{C_v}$, $\boldsymbol{\gamma C_{st}}$ and $\boldsymbol{\gamma C_{jk}}$ denote the specific heat of supercritical helium at the constant volume, the heat capacity of superconducting strands, the heat capacity of conduit, respectively. $\boldsymbol{k_{st}}$ and $\boldsymbol{k_{jk}}$ are the thermal conductivity of superconducting strands and conduit, respectively. $\boldsymbol{d_h}$ is the thermal hydraulic diameter. $\boldsymbol{p_{jk}}$ is the wetted parameters of conduit. $\boldsymbol{A_c}$, $\boldsymbol{A_{he}}$ and $\boldsymbol{A_{jk}}$ are the total cross sectional area of superconducting strands and pure copper strands, the cross sectional area of conduit and the total cross sectional area of helium, respectively. $\boldsymbol{h_{jk}}$ is the heat transfer coefficient between the helium and conduit. $\boldsymbol{f}$ is the friction factor. $\boldsymbol{q_{jjk}}$, $\boldsymbol{q_{jst}}$, $\boldsymbol{q_{djk}}$ and $\boldsymbol{q_{dst}}$ are the Joule heat power and disturbance power in the conduit and superconducting strands, respectively. The equation includes the convection and diffusion terms. The coefficient matrix $\boldsymbol{K}$ is related to the thermal diffusion term of superconducting strands and conduit. The coefficient matrix $\boldsymbol{A}$ is connected to the convection terms of the supercritical helium. The source term includes the external heat disturbance and Joule heating generation. The disturbance power q_d is with the Gaussian distribution. The Joule heating power depends on the current sharing temperature (T_{sh}) and critical temperature (T_C) of superconducting strands.

III. NUMERICAL SOLUTION AND ADAPTIVE MESH

The numerical solution of the governing equation by the central differences is unstable. It is necessary to add the artificial diffusion term to stabilize the oscillation of solution. To avoid the adding artificial viscosity term which influences the accuracy of solution, the finite volume method of upwind scheme, which can stabilize the numerical solution of hyperbolic convection dominated flow problem, is applied to discretize the space terms on the staggered mesh [3,4]. The control volumes for the velocity of supercritical helium are staggered relative to the control volumes of density and temperature of supercritical helium. The centers of the control volumes for velocity are always the boundaries of the control volumes for the density and temperature even when the control volume varies. The centers of the control volumes for the velocity located at the inlet and outlet ends of CICC are the half volume with the elements. The first and final control volumes of density and temperature are located at the inlet and outlet boundaries with full elements. According to the finite volume methods of upwind scheme, the equations for the continuity, momentum and energy conservation of supercritical helium and conduit are discretized.

The time term of the variable ψ is approximated by using (n+θ) where θ is varied in the range of (0-1). For θ=1/2, the time integral has second-order accuracy.

$$\left.\frac{\partial\psi}{\partial t}\right|^{n+\theta}=(1-\theta)\left.\frac{\partial\psi}{\partial t}\right|^{n}+\theta\left.\frac{\partial\psi}{\partial t}\right|^{n+1}=\frac{\psi^{n+1}-\psi^{n}}{\Delta t} \tag{2}$$

After the discretization of space and time terms, a nonlinear problem with full implicit time for $0<\theta\le 1$ can be obtained. However, the fine mesh should be taken to obtain the converged numerical solution in the long length of CICC conductor that is used in the superconducting Tokamak magnets. Therefore, a lot of control volumes must be employed and the iterative method to solve above equation needs much more CPU time. In order to overcome the problem, the coefficients of the discretized equations at the $(n+1)$ th time are calculated by n th time parameters. Before assembly of the matrix structure for the linear system, it must note the order of the discretized equation, such as continuity, momentum, energy and conduit. This is the reason why the main diagonal term over the other elements in the matrix structure should be dominated to get stable solution. The coefficient matrix of discretized equations has a large-scale sparse structure. A compressible storage scheme for the sparse problem is applied to solve the equation in IMSL [5].

When the superconducting strands in the CICC quench, the strong heat coupling between the supercritical helium and superconducting strands may induce the discontinuity of the temperature and density at the normal zone front [6]. In order to capture the short normal zone, the fine mesh at the front region of normal zone is essential. The typical mesh size should be at the region of 1-5 mm [7], depending on the material characteristics. Because the length of CICC for the superconducting magnet is very long, typical length is over 10^2-10^3 m, therefore, a scheme of adaptive mesh is demanded. The new mesh is generated with the finest mesh at the normal zone front and it is independent of the old mesh. The scheme uses an algebraic transformation of the equal interval coordinate (ζ) to non-equal interval coordinate ($\boldsymbol{x}$). The basic transformation is given as [8]:

$$x=x_q\left(1+\frac{sinh[\tau(\varsigma-\delta)]}{sinh(\tau\delta)}\right),\quad \delta=\frac{1}{2\tau}ln\left(\frac{1+(e^{\tau}-1)x_q}{1+(e^{-\tau}-1)x_q}\right) \tag{3}$$

In the transformation, x_q is the normal zone front coordinate, τ is the stretching parameter which varies from zero to large values and generates the maximum refinement near x_q. The smoothly varying mesh size is controlled by setting the maximum mesh size, $\Delta x_{\max}$, and minimum mesh size, $\Delta x_{\min}$.

VI. ERROR ANALYSIS AND APPLIED TO KSTAR MAGNETS

Based on the above discussions of numerical method, the code, QSAIT, has been developed for the thermal hydraulic analysis. Verification of the numerical code QSAIT has been performed by comparison with experimental measurements and numerical results of other codes [9]. In generally, the CPU time used by the code QSAIT are about 40 minutes on an ALPHA PC computer for 2 s quench simulation. It is shorter that that of the QUENCHER about 50 min of CPU time on a VAX Station 4000/90.

TABLE I
TF AND PF CONDUCTOR PARAMETERS OF KSTAR

Parameter	Units	PF1-5	PF6-7	TF
Conductor		Nb_3Sn	NbTi	Nb_3Sn
Conduit		Incoloy 908	316LN	Incoloy 908
Cu/Noncu		1.5:1	3.5:1	1.5:1
Aconduit	(mm^2)	175.6	175.6	244.6
D_{strand}	(mm)	0.78	0.78	0.78
$n_{strands}$		360	360	486
$n_{custrands}$		120	120	162
$h_{conduit}$	(mm)	22.3	22.3	25.65
$w_{conduit}$	(mm)	22.3	22.3	25.65
$t_{conduit}$	(mm)	2.41	2.41	2.86
A_{cu}	(mm^2)	126.1	146.5	170.3
A_{noncu}	(mm^2)	45.8	25.5	62.9
A_{Hecond}	(mm^2)	111.4	111.4	126.9
Lstrand	(km)	1625.8	1563	3194
Lcable	(km)	6.82	7.33	9.86
Mscstrand	(tons)	7.0	7.5	13.4
Ncoils		10	4	16
J_{noncu}	(A/mm^2)	544.2	641.8	540

In order to show the some dependence of the parameters, such as maximum quench pressure (P), maximum temperature rise in strands (T_{st})and normal zone length ($2x_q$) on the step-size of space and the integration time step-size, a 5 s quench simulation in TF conductor of KSTAR, which the main parameters are listed in Table I, is performed with various Δx values in range of 0.5-100 mm with constant time step-size of 0.1 ms. The inlet and outlet pressures and inlet temperature are 5 atm, 3 atm and 5 K, respectively. Fig.1 shows the relative errors profiles of maximum pressure, normal zone length and normal zone voltage for initial disturbance lengths of 2 and 3 m, where the values calculated by Δt =0.1 ms and dx_{min} =3.5 mm are used as the references [10]. From Fig. 1, the relative error values of the pressure and normal zone terminal voltage are less than 10 % during the variations of the dx_{min} from 3.5 to 10 mm. While the dx_{min} is increased from 3.5 to 20 mm, the relative errors are increased over 20 %. It shows that the normal zone length is more sensitive to the space stepsize [11]. The profiles of maximum temperature versus the space step-sizes are illustrated in Fig.2. Generally, with the increment of space step-size, the maximum temperature of hot-spot is decreased. The short normal zone length leads to a high hot spot temperature rise in CICC. Fig.3 shows the relative values of maximum pressure and normal zone length versus the time stepsize during the space stepsize of dx_{min} = 3.5 mm for the initial disturbance lengths of 3 and 5 m. From the Fig. 1, 2 and 3, it is difficult to obtain the converged solution for the shorter disturbance. This is because the shorter disturbance can generate more steep temperature profiles in the normal zone front. In order to obtain converged solution and to capture the contact discontinuity of temperature and density, the small stepsizes for the space and time are needed.

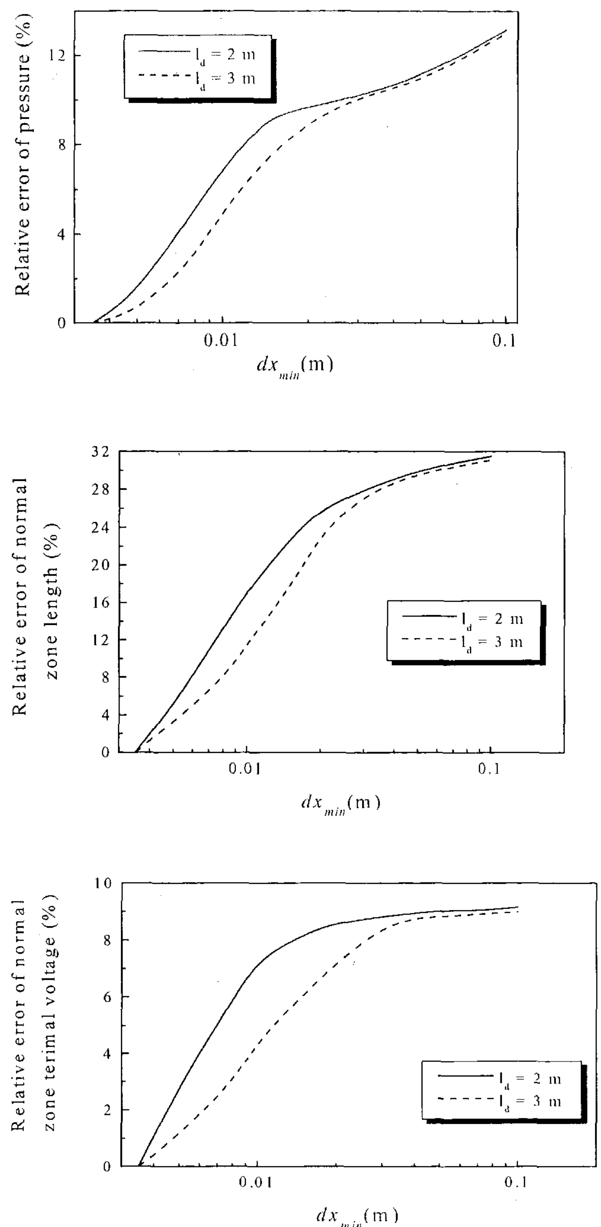

Fig. 1 Relative error profiles of maximum pressure, normal zone length and voltage of normal zone terminate versus space step. The values calculated by Δt =0.1 ms and dx_{min} =3.5 mm are used as the reference values.

The code is also applied to analysis of the quench in the KSTAR PF and TF magnets. The operating conditions of inlet pressure and temperature are 0.4 MPa for the PF1-4 and 0.5 MPa for the PF5-7, and 5 K for all the PF magnets. The operating conditions of inlet pressure and temperature are 0.5 MPa and 5 K for the TF, respectively. We assume the zero mass flow in the conductors. In the study, the quench detection is set. The delaying time is 1 s and the decaying time constant of 3.56 s. Two scenarios are considered: long initial normal zone and short normal zone. Generally, the long initial zone corresponds to the whole high field regions quench simultaneously, and the short initial normal zone is a minimum propagation zone. The disturbance with length of 3 m and 0.65 m and the duration time of 10 ms assumes to be applied to the conductor. The peak operating current (I) and peak field (B) are listed in Table II. The quench simulation results for the KSTAR magnets are listed in the Table II. It shows the maximum temperature of hot-spot and peak pressure are located at the PF1. The maximum temperature and pressure are lesser than the design limitation of 150 K and 5 MPa.

TABLE II
PARAMETERS OF QUENCH IN PF CONDUCTOR OF KSTAR AFTER 5 s (Disturbance length L_{1d} = 0.65 m, L_{2d} =3 m)

		PF1	PF2	PF3	PF4	PF5	PF6	PF7	TF
I	KA	26.45	25.5	13.1	25.3	26.26	0.76	1.71	35.2
B	T	6.72	7.58	7.58	7.22	4.3	1.27	1.69	7.8
Presure	L_{1d}	7.92	6.63	6.66	6.11	6.02	5.3	6.95	8.98
(atm)	L_{2d}	19.4	13.9	14.2	7.1	7.7	5.95	12.4	17.2
T_{st}(K)	L_{1d}	81.3	58.1	58.2	54.9	34.2	12.9	28.9	85.6
	L_{2d}	74.2	54.1	54.2	50.7	32.0	12.8	25.5	74.6
$2x_q$ (m)	L_{1d}	10.2	7.53	7.54	6.5	2.58	0.87	5.2	8.5
	L_{2d}	1.28	0.5	0.5	0.18	0.08	0.04	0.15	2.4
Voltage	L_{1d}	0.59	0.24	0.24	0.18	0.03	0.003	0.05	0.78
(V)	L_{2d}	1.28	0.5	0.5	0.18	0.08	0.04	0.15	2.4
T_{jk} (K)	L_{1d}	66.1	51.3	51.4	48.9	33.0	12.8	28.1	69.0
	L_{2d}	61.6	48.3	48.4	45.8	30.8	12.7	24.9	65.6

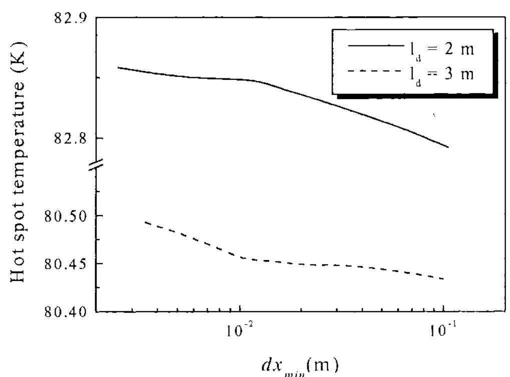

Fig. 2 Hot-spot temperature profiles versus space interval, we assume the time stepsize to be kept constant Δt = 0.1 ms.

VII. Conclusions

A numerical method has been developed based on the finite volume method of upwind scheme. The model uses the high heat transfer approximation between the superconducting strands and supercritical helium. The code, QSAIT, has following characteristics: (1) The supercritical helium pressure, normal zone length are sensitive to the stepsizes of space and time. (2) Using staggered mesh scheme and upwind finite volume method can circumvent the artificial viscosity. (3) The stable solutions can be obtained at larger time step-size, i.e. the limiting time step-size can be eliminated by the θ-implicit scheme. (4)The adaptive scheme installed in the code reduces the number of control volumes so as to obtain converged solution.

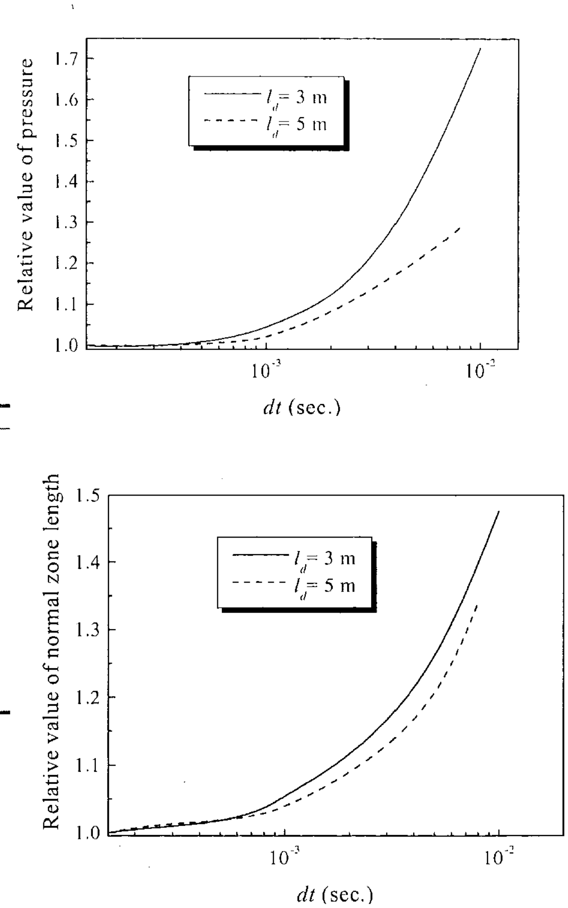

Fig. 3 Relative values of maximum pressure, normal zone length versus time step. The values calculated by Δt =0.1 ms and dx =3.5 mm is employed as the reference values.

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