A Consistent Description of Scaling Law for Flux Pinning in Nb_3Sn Strands Based on the Kramer Model

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***Abstract*—There are a lot of experimental reports on the scaling of flux pinning in the form of $F = F_m b^{1/2}(1-b)^2$, with $b = B/B_{c2}$. The temperature dependence of F_m is approximately proportional to $B_{c2}^{5/2}$, whereas the strain dependence of F_m is reported to be proportional to the upper critical field Bc2. In this work, we re-analyze our previous data with the Kramer model including the pin-breaking dynamic pinning force (F_p) for a low field region. It is shown that the extrapolated upper critical field B^*_{c2}, strongly depend on the ratio between the mean of the parameter K_p for F_p ($< K_p >$) and the parameter K_s for the flux line lattice shearing pinning force F_s. It is found that the strain dependence of F_m at 4.2 K is approximately proportional to $(B^*_{c2})^{1.5}$. We further compare the data with the prediction of our recent scaling theory based on Eliashberg theory of strongly coupled superconductors. It is shown that the strain dependence of F_m at 4.2 K is proportional to $B_{c2}^{5/2}\kappa^{-2}$, consistent with the temperature dependence of F_m. Moreover, this model agrees reasonably well even with the data in a high compressive strain region (< -0.8%).**

***Index Terms*—Eliashberg theory, scaling of flux pinning, strain dependence, the Kramer model.**

I. Introduction

A SCALING law of flux pinning for a number of Nb-based alloy was first proposed by Fietz and Webb as a function of the reduce field $b = B/B_{c2}$ [1]. It was reported that the pinning force F can be written as, $F = I_c \times B = F_m(T)f(b) = F_m b^p(1-b)^q$, with $p \approx 0.5$, $q \approx 2$. The temperature dependence of the pinning force maximum F_m is approximately proportional to $B_{c2}^{5/2}$. Later, Ekin found that the strain dependence of pinning force can be written with the same functional form of $f(b)$ but F_m is proportional to B_{c2}, especially for Nb_3Sn strands [2]. This discrepancy is one of the major problems for a consistent understanding of scaling laws for flux pinning.

In an attempt to understand the scaling law of Fietz and Webb, Kramer proposed a flux line lattice (FLL) shearing model, where the pinning force is written as a function of the upper critical field B_{c2} and the Ginzburg-Landau parameter κ, $F = C \cdot B_{c2}^{5/2}\kappa^{-2}b^p(1-b)^q$, $(p = 0.5, q = 2)$ [3]. The relation $I_c \times B \propto b^{1/2}(1-b)^2$ or $I_c^{1/2} \times B^{1/4} \propto 1 - B/B_{c2}$, is widely used for the extrapolation of B_{c2} from the critical current measurement. However, there are reports on the inconsistency between the actual and the extrapolated critical fields [4], [5] and $p \approx 0.5$, $q \approx 2$ does not hold even for some of the Nb_3Sn strands reported [5]. In the strain scaling law of Ekin, F_m is proportional to the extrapolated B^*_{c2} [2], [5].

In the original work of Kramer, however, the pin-breaking dynamic pinning force F_p is also considered. The competition between F_p and the FLL shearing dynamic pinning force F_s, determines the overall shape and maximum position of $f(b)$. In this work, we re-examine the Kramer model including F_p, and compare it with the experimental results (Section II). In particular, our previous critical current measurement data as a function of strain at 4.2 K are re-analyzed based on the original Kramer model (Section III).

II. Comparison of the Kramer Model Including the Pin-Breaking Dynamic Pinning Force With Experiments

At high reduced fields, Kramer assumed that strong pins do not break and the FLL shear plastically. He showed that for a set of planar pins, the FLL shearing dynamic pinning force F_{s}, can be written as, $F_s = K_s b^{1/2}(1-b)^2$, with $K_s \propto B_{c2}^{5/2}\kappa^{-2}$. For Nb_3Sn strands, it was reported that grain boundaries are the major pinning centers [6], [13], and the FLL shearing model is consistent with the temperature scaling law of Fietz and Webb. At low reduced fields, however, this synchronous plastic motion of FLL is implausible and it is considered that flux motion is dominated by unpinning of line pins. Taking into account the increase of pinning due to FLL shear, Kramer argued that the pin-breaking dynamic pinning force F_p is, $F_p = K_p b^{1/2}(1-b)^{-2}$, with $K_p \propto B_{c2}^{5/2}\kappa^{-2}$. The maximum pinning force will be reached when $F_p \approx F_s$. For the de-pinning of individual line pins, we need to consider the statistical distribution of pinning strengths. If we assume a Poisson distribution, then the overall pinning force can be written as [3]

$$F = I_c \times B = F_m \cdot f(b) = C \cdot B_{c2}^{\frac{5}{2}}\kappa^{-2} \cdot f(b)$$

with

$$f(b) = b^{\frac{1}{2}} \lfloor g(1-b)^{-2}\left\{1 - \left(\frac{1+(1-b)^4}{g}\right) \cdot e^{\frac{-(1-b)^4}{g}}\right\} + (1-b)^2 \cdot e^{\frac{-(1-b)^4}{g}} \rfloor, \quad g = \frac{\langle K_p \rangle}{K_s}. \quad (1)$$

The normalized pinning force $F/K_s (= f(b))$, strongly depends on the variation of the ratio between the mean K_p and K_s (factor $g = \langle K_p \rangle / K_s$), as shown in Fig. 1 (thin solid lines). As the average pinning strength $\langle K_p \rangle$ increases, the peak in $f(b)$ shifts to lower b and the peak height increases, and finally $f(b)$

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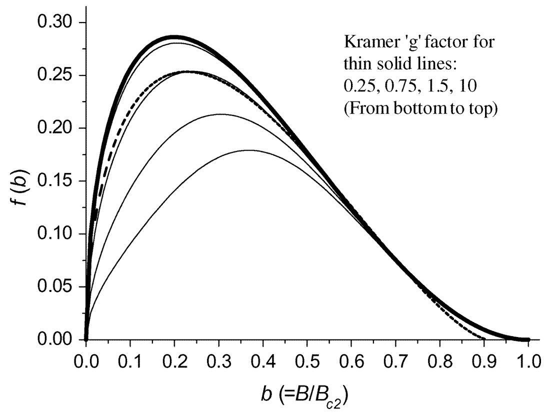

Fig. 1. Pinning force function $f(b)$ as a function of b calculated with (1) for several values of 'g' (thin solid lines). A thick solid line corresponds to the usual Kramer FLL shearing model, $f(b) = b1/2(1-b)^2$. The normalized function, $f(b) = 0.78(b/0.91)^{0.5}(1-b/0.91)^{1.5}$, is represented as a dotted line.

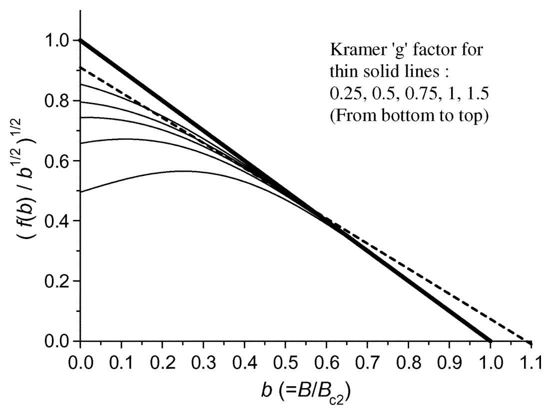

Fig. 2. The Kramer plots for several values of 'g' (thin solid lines). $(f(b)/b^{1/2})^{1/2}$ is proportional to $I^{1/2}B^{1/4}$. A thick solid line corresponds to the usual Kramer FLL shearing model, $f(b) = b^{1/2}(1-b)^2$. For g=1 case, $(f(b)/b^{1/2})^{1/2}$ within range $0.27 < b < 0.50$, is fitted linearly (dotted line).

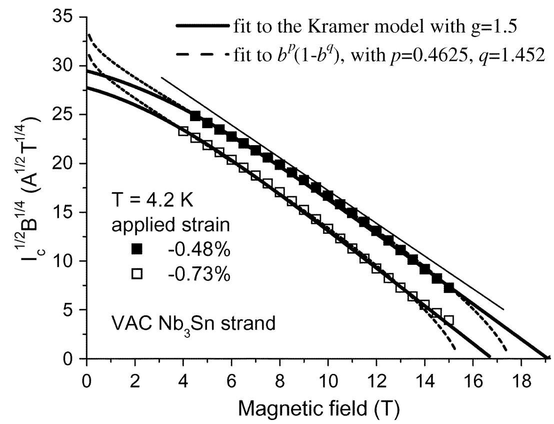

Fig. 3. The Kramer plot of VAC Nb_3Sn strand [8]. Dotted lines are fitting results with a function $f(b) = b^p(1-b)^q (p = 0.4652, q = 1.452)$. Thick solid lines are obtained from fits to (1), with $g = 1.5$.

merges into $b^{1/2}(1-b)^2$, the normalized pinning force of the usual Kramer FLL shearing model (thick solid line in Fig. 1). Using the conventional Kramer FLL shearing model implicitly means that we are assuming a large average pinning strength $\langle K_p \rangle$ (larger than 10 K_s). (Similar results were observed for other distribution functions as well [3].)

It was reported that increasing the grain size in Nb_3Sn can decrease F_m to less than $1/10$ [6], [7]. If $\langle K_p \rangle$ is comparable to K_s in an actual strand, the usual extrapolation method can lead to an overestimation of B_{c2} as can be seen in Fig. 2. The Kramer plots for various values of $g$are presented in Fig. 2. $(f(b)b^{1/2})^{1/2}$ is proportional to $I^{1/2}B^{1/4}$. For intermediate values of g (for example, 0.75, 1, 1.5 in Fig. 2), $(f(b)b^{1/2})^{1/2}$ are slightly concaved at low reduced field. B_{c2} of an actual strand is $\sim$ 30 T and the critical current is usually measured with an applied field ranging from 8 T to 15 T. In this case, b varies from 0.27 to 0.5. If B_{c2} is obtained from the conventional linear extrapolation with data within this range, the extrapolated $B_{c2}^{\sim}$ will be strongly affected by the value of g. For example, the deviation can be as high as 9% with $g = 1$ (dotted line in Fig. 2).

With an applied strain, B_{c2} is decreased and the critical current measurement for a wide range of b becomes possible. The Kramer plots obtained from the critical current measurement with the applied strain of -0.43% and -0.73% for a VAC strand, reported by Taylor *et al.* [8], are shown in Fig. 3. It is obvious that the usual linear extrapolation does not give a reasonable fit to the data (see the thin linear guideline in Fig. 3). Taylor analyzed the data with a function $f(b) = bp(1-b)^q$, and best fits were obtained for $p = 0.4625$, $q = 1.452$ [8] (dotted lines in Fig. 3). On the other hand, the Kramer model with the factor $g = 1.5$, also give reasonable fit to the data as can be seen in Fig. 3 (thick solid lines).

One of obvious differences between the two extrapolation methods is that the extrapolated B_{c2}^* from the Kramer model with the factor $g = 1.5$ is $\sim$ 9% larger than that from a fit to $f(b) = b^p(1-b)^q$. From the critical current measurement at high field up to 24 T, Ekin reported that the data can be well describe by a function $f(b) = b^p(1-b)^q$, with $p = 0.45$, $q = 1.4$, for 2 wt.% Ta added ternary Nb_3Sn strand, especially near B_{c2} [5]. Recently, Suenaga *et al.* observed the irreversibility field for a wide range of temperature from the magnetization measurement of 2 at.% Ti added ternary Nb_3Sn strand [9]. They further showed that the irreversibility field can be understood from the FLL melting theory of Houghton *et al.* [10]. If FLL melting occurs in Nb_3Sn strands, the Kramer FLL shearing model cannot be applied at fields near B_{c2}.

If we neglect the region near B_{c2}, good similarity can be found between the Kramer model with the factor g and a function $f(b) = b^p(1-b^q$, with arbitrary p, q. For example, $p = 0.5$, $q = 1.5$, was reported for 1.85 at.% Ti added ternary Nb_3Sn strand [5]. In Fig. 1, $f(b) = b^p(1-b)^q$, with $p = 0.5$, $q = 1.5$, plotted as a dotted line, shows good resemblance with the solid line calculated using the Kramer model with the factor $g = 1.5$ for $0.2 < b < 0.75$ (for comparison, 9% reduced value of B_{c2} was used). However, the merit of using the Kramer model with the factor g is clear. When the Kramer model is used, the relation between F_m and the superconducting parameters is exactly given, $F_m \propto B_{c2}^{5/2}\kappa^{-2}$. And the material properties can be more easily incorporated into the model. The factor g is related with the pinning properties of the sample, which are related with the grain size, the addition of third element and so on. Based on these ideas, our previous data at 4.2 K are re-analyzed in the

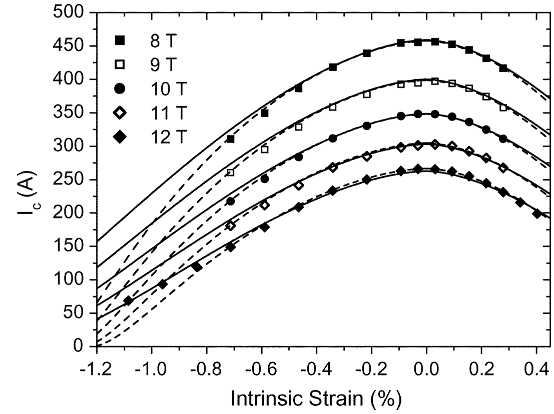

Fig. 4. The critical current as a function of intrinsic strain for KAT Nb_3Sn strand type A at 4.2 K. Dotted lines are calculated with the strain scaling law of Ekin, and solid lines are calculated with (1), (2) and (3).

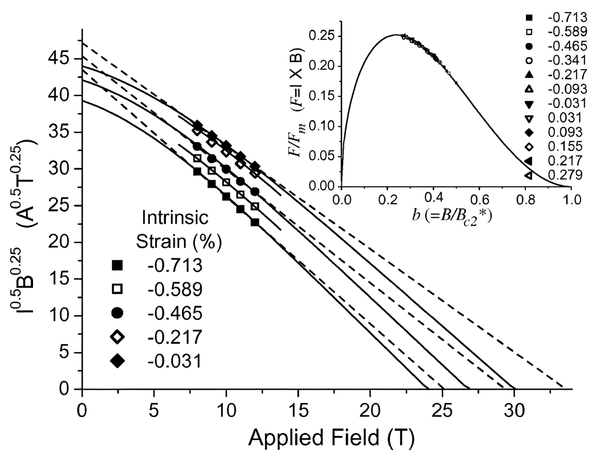

Fig. 5. The Kramer plots for KAT type A. Solid lines are fitting results with (1), $g = 1.5$, and dotted lines are the conventional linear extrapolation. Inset: Normalized pinning force versus reduced field.

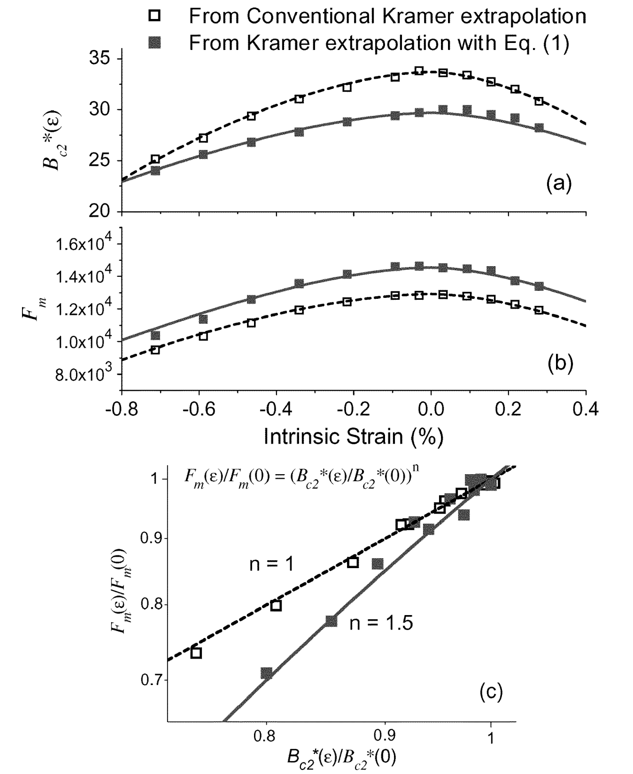

Fig. 6. (a) The extrapolated upper critical field B_{c2}^{*} as a function of intrinsic strain. A dotted line is calculated with the scaling law of Ekin and a solid line is calculated with (2). (b) F_m as a function of intrinsic strain. A dotted line is calculated with the scaling law of Ekin and a solid line is calculated with (3). (c) Log-log plots of the normalized F_m versus the normalized B_{c2}^{*}.

next section, to see the validity of the Kramer model for the strain dependence of pinning force.

III. The Strain Dependence of Pinning Force at 4.2 K

The strain dependence of the critical current at 4.2 K for KAT Nb_3Sn strand is shown in Fig. 4 [11]. The Kramer plots are presented in Fig. 5. Dotted lines are the conventional linear extrapolations, and solid lines are fits to (1), the Kramer model including the pin-breaking dynamic pinning force. Here, the factor $g = 1.5$ is used. KAT Nb_3Sn strands are ternary compound with 1.5 $\sim$ 3 wt.% Ti added. In the previous section, it was shown that the 1.85 at.% Ti added ternary Nb_3Sn strand can be well described with g = 1.5. When the intrinsic strain of the strand is -0.031%, the extrapolated upper critical field $B_{c2}^{\sim}$ from the conventional method is $\sim$ 33.7 T, while using the new extrapolation, B_{c2}^{*} is $\sim$ 29.7 T. This discrepancy is decreased with decreasing the intrinsic strain as can be more clearly seen in Fig. 6(a).

Fig. 6(a) shows the strain dependence of B_{c2}^{*}. Open squares are obtained from the conventional extrapolation and solid squares from the new extrapolation method. It was reported that B_{c2}^{*} at 4.2 K can be expressed as, $B_{c2}^{*}(\varepsilon) = B_{c2}^{*}(0)(1 - a|\varepsilon|^{1.7})$ [2]. A dotted line in Fig. 6(a) is a fit to this equation. For compressive strain, the strain dependency coefficient a is 1150 and for tensile strain, 1700. If B_{c2}^{*} from the new extrapolation method is fitted using the same equation, the strain dependency coefficient a decreases noticeably to 860, 1200, for compressive and tensile strain, respectively. Logarithmic plots of the normalized F_m as a function of the normalized B_{c2}^{*} are shown in Fig. 6(c). When B_{c2}^{*} and F_m are extracted from the conventional Kramer extrapolation, F_m is proportional to B_{c2}^{*} (open squares), as was reported previously [2]. However, when (1) is used for the extrapolation, F_m is proportional to $(B_{c2}^{*})^{1.5}$.

In order to clarify the validity of the Kramer model, we further need to examine the relation, $F_m \propto B_{c2}^{5/2}\kappa^{-2}$. The strain dependence of κ, is not yet known empirically. On the other hand, from the numerical calculation of Eliashberg equations for

TABLE I
Parameters for the Strain Scaling of Ekin

n	B_{c2}*(0)	p	q	a Compressive strain	a Tensile strain
1	33.8 T	0.5	2	1150	1700

TABLE II
Parameters for the Strain Scaling Proposed in this Work

g	B_{c2}*(0)	Compressive strain		Tensile strain	
		β	γ	β	γ
1.5	29.7 T	410	480	580	680

strongly-coupled superconductors, we recently showed that the strain dependence of the thermodynamic critical field B_c, and the Ginzburg-Landau parameter κ, can be written with the same functional form for the strain dependence of the transition temperature T_c [12]. If we neglect the temperature dependence, the strain dependence of superconducting parameters can be written as

$$\begin{aligned} T_c(\varepsilon) &= T_c(0)(1-\alpha|\varepsilon|^{1.7}) \\ B_c(\varepsilon) &= B_c(0)(1-\beta|\varepsilon|^{1.7}) \\ \kappa(\varepsilon) &= \kappa(0)(1-\gamma|\varepsilon|^{1.7}) \\ B_{c2}(\varepsilon) &= \sqrt{2}\cdot B_c(\varepsilon)\kappa(\varepsilon) = B_{c2}(0)(1-\beta|\varepsilon|^{1.7})(1-\gamma|\varepsilon|^{1.7}). \end{aligned} \tag{2}$$

It was reported that there is a relation between $T_c(\varepsilon)$ and $B^*_{c2}(\varepsilon) = B^*_{c2}(0)(1-a|\varepsilon|^{1.7})$, which can be written as, $T_c(\varepsilon)/T_c(0) = (B^*_{c2}(\varepsilon)/B^*_{c2}(0))^{1/w}$ with $w \approx 3$ (or if we put it in other way, $\alpha \approx a/3$) [2]. The relations among α, β, and γ obtained from the microscopic theory can be approximately written as, $\beta \approx 4\alpha/3$, $\gamma \approx 5\alpha/3$ [12].

The solid line in Fig. 6(a) is calculated with (2), $B_{c2}(\varepsilon) = B_{c2}(0)(1-\beta|\varepsilon|^{1.7})(1-\gamma|\varepsilon|^{1.7})$. With the new extrapolation method using (1), the strain dependency coefficient a of 860 is obtained for the compressive strain. When $a = 860$, the value of β and γ are numerical obtained from the microscopic theory, as 410, 480. For the tensile strain ($a = 1200$), we assume the same ratio, and the resulting β and γ are 580, 680. These values of β and γ reproduce the data reasonable well as can be seen in Fig. 6(a).

If we apply the above strain dependence of superconducting parameters to the Kramer model, the stain dependence of F_m is expressed as

$$\begin{aligned} F_m(\varepsilon) &= C\cdot B_{c2}(\varepsilon)^{\frac{5}{2}}\kappa(\varepsilon)^{-2} \\ &= F_m(0)(1-\beta|\varepsilon|^{1.7})^{\frac{5}{2}}(1-\gamma|\varepsilon|^{1.7})^{\frac{1}{2}} \end{aligned} \tag{3}$$

The solid line in Fig. 6(b), which shows good agreement with the data, is calculated with (3) using the same coefficients β and γ. The empirical relation, $F_m \propto (B^*_{c2})^{1.5}$ can be understood as originated from the difference in the strain dependency coefficient β and γ for B_c and κ. We suggest that the strain dependence of pinning force in Nb_3Sn can be understood on the basis of the Kramer model including the pin-breaking dynamic pinning force together with the strain dependence of superconducting parameters from the microscopic theory.

The dotted lines in Fig. 4 are calculated with the strain scaling law of Ekin, and the solid lines are calculated with (1), (2), and (3). The scaling parameters used are summarized in Tables I and II. For a low strain region (-0.6%< ε < 0.4%), the critical current data are in good agreement with both scaling laws. However, at high compressive strain region (ε< -0.8%), only the strain scaling law based on the Kramer model and Eliashberg theory gives a reasonable fit to the data. Recently, there is a controversy on the accuracy of the Ekin-Summer's scaling law at high compressive strain region (ε<-0.8%) [13].

For the ternary Nb_3Sn strand studied in this work, the new scaling law based on the Kramer model and Eliashberg theory explains the un-expected large value of the extrapolated B^*_{c2} (33.7 T from the conventional extrapolation) and the critical currents larger than the prediction of the strain scaling law of Ekin at high compressive strain region (ε< -0.8%), with a reasonable number of parameters. In order to validate the Kramer model, further comparative studies for various kinds of strands are needed.

IV. Conclusion

In summary, we showed that the reported inconsistency between the actual and the extrapolated upper critical field can be resolved with the original Kramer model, where the pin-breaking dynamic pinning force is included. For the Ti added ternary Nb_3Sn strand, the pinning function $f(b)$ from the Kramer model with $g = 1.5$ is comparable with the empirically reported $f(b) = b^p(1-b)^q$, with $p \approx 0.5$, $q \approx 1.5$. $f(b)$ from the Kramer model with the factor g is used further for the extrapolation of B_{c2} and F_m for the ternary KAT Nb_3Sn strand at 4.2 K, and it is shown that F_m is proportional to $(B^*_{c2})^{1.5}$. By using the strain dependence of superconducting parameters from the microscopic theory, we showed that a consistent description of scaling for flux pinning is possible by the Kramer model. The scaling law proposed in this work, successfully explained the strain dependence of the critical current at 4.2 K even at high compressive strain region (ε<-0.8%). We expect that the proposed scaling law will be useful for application such as the design optimization of large scale magnets.

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