A scaling law for the critical current of Nb_3Sn stands based on strong-coupling theory of superconductivity

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We study the transition temperature T_c, the thermodynamic critical field B_c, and the upper critical field B_{c2} of Nb_3Sn with Eliashberg theory of strongly coupled superconductors using the Einstein spectrum $\alpha^2(\omega)F(\omega)=\lambda\langle\omega^2\rangle^{1/2}\delta(\omega-\langle\omega^2\rangle^{1/2})$. The strain dependences of $\lambda(\varepsilon)$ and $\langle\omega^2\rangle^{1/2}(\varepsilon)$ are introduced from the empirical strain dependence of $T_c(\varepsilon)$ for three model cases. It is found that the empirical relation $T_c(\varepsilon)/T_c(0)=[B_{c2}(4.2\text{ K},\varepsilon)/B_{c2}(4.2\text{ K},0)]^{1/w}$ ($w\approx 3$) is mainly due to the low-energy-phonon mode softening. We derive analytic expressions for the strain and temperature dependences of $B_c(T,\varepsilon)$ and $B_{c2}(T,\varepsilon)$ and the Ginzburg-Landau parameter $\kappa(T,\varepsilon)$ from the numerical calculation results. The Summers refinement on the temperature dependence of $\kappa(T)$ shows deviation from our calculation results. We propose a unified scaling law of flux pinning in Nb_3Sn strands in the form of the Kramer model with the analytic expressions of $B_{c2}(T,\varepsilon)$ and $\kappa(T,\varepsilon)$ derived in this work. It is shown that the proposed scaling law gives a reasonable fit to the reported data with only eight fitting parameters. © *2006 American Institute of Physics*.
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I. INTRODUCTION

For large scale superconducting Tokamak magnets, such as International Thermonuclear Experimental Reactor (ITER), Nb_3Sn wires are widely used in the form of cable-in-conduit conductor (CICC). Due to the thermal contraction difference between constituent materials, the compressive stain of Nb_3Sn strands in CICC at the operation temperature can be as high as $\sim 0.8\%$.[1] At 4.2 K and 12 T, the critical current could decrease more than 50% with the application of -0.8% strain.[2–6] The critical current of type-II superconductor is determined by a balance between the Lorentz force and the pinning force ($F_p=I_c\times B$).[7] A scaling law of the pinning force was reported by Fietz and Webb.[8] They found that the pinning force can be written as a function of the superconducting parameters, such as the upper critical field B_{c2} and the Ginzburg-Landau parameter κ. The pinning force is proportional to ${B_{c2}}^{\nu}\kappa^{-\gamma}\cdot b^p(1-b)^q$, with $2\leqslant\nu\leqslant 3$, $1<\gamma<3$, $p\approx 0.5$, and $q\approx 2$, where b is the reduced field defined as $b=B/B_{c2}$. A scaling law of flux pinning under axial strain in a superconducting wire was further reported by Ekin.[9] For Nb_3Sn strands, the pinning force at 4.2 K is proportional to ${B_{c2}}^{n}\cdot b^p(1-b)^q$, with $n\approx 1$, $p\approx 0.5$, and $q\approx 2$. It was shown that the strain dependence of $B_{c2}(4.2\text{ K},\varepsilon)$ can be written as $B_{c2}(4.2\text{ K},\varepsilon)=B_{c2}(4.2\text{ K},0)(1-a|\varepsilon|^{1.7})$. He also reported that the strain dependence of the critical temperature $T_c(\varepsilon)$ is related to $B_{c2}(4.2\text{ K},\varepsilon)$ as $T_c(\varepsilon)/T_c(0)=[B_{c2}(4.2\text{ K},\varepsilon)/B_{c2}(4.2\text{ K},0)]^{1/w}$, with $w\approx 3$.

On a proper unified strain and temperature scaling law for Nb_3Sn strands, there is still a controversy.[2,5,10–12] In the temperature scaling law of Fietz and Webb, the pinning force maximum is proportional to ${B_{c2}}^{2/5}\kappa^{-2}$, while in the strain scaling law of Ekin, it is propotional to B_{c2} for Nb_3Sn strands. Summers *et al.* proposed an expression for the temperature dependence of $\kappa(T)$.[13] Together with the strain dependences of the superconducting parameters reported by Ekin, an empirical formula for the critical current as a function of field, temperature, and strain was derived. The unified scaling law of Summers was further elaborated by ten Haken *et al.*[2] Three-dimensional deviatoric strain dependences of the superconducting parameters were considered. Recently, Keys *et al.* proposed fourth-order polynomial fits for the strain dependences of the superconducting parameters from their extensive critical current measurements and suggested that the pinning force can be written in the form of the Kramer model.[5]

To understand the scaling law for the critical current in type-II superconductor, we need a better understanding for the pinning properties. However, the strain and temperature dependences of the superconducting parameters should be understood from a microscopic theory. Markiewicz, recently, tried to understand the strain dependence of $T_c(\varepsilon)$ on the basis of strong-coupling theory with his elastic stiffness model but the strain and temperature dependences of $B_{c2}(T,\varepsilon)$ were not calculated in his work.[14] Here we try to understand the relation $T_c(\varepsilon)/T_c(0)=[B_{c2}(4.2\text{ K},\varepsilon)/B_{c2}(4.2\text{ K},0)]^{1/w}$ ($w\approx 3$) within the framework of Eliashberg strong-coupling theory.[15,16] In the strong-coupling theory, the superconducting properties can be parameterized by the coupling constant λ and the charateristic phonon frequency $\langle\omega^2\rangle^{1/2}$, which can be obtained from the electron-phonon spectrum density function $\alpha^2(\omega)F(\omega)$.[17–20] In this work, three model cases for the variation of λ and $\langle\omega^2\rangle^{1/2}$ are considered, as explained in Sec. II. The transition temperature and the thermodynamic critical field are numerically calculated from Eliashberg equations. The strain dependences of λ

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and $\langle\omega^2\rangle^{1/2}$ for each case are obtained from the strain dependence of $T_c(\varepsilon)$. In Sec. III, it is shown that the relation $T_c(\varepsilon)/T_c(0)=[B_{c2}(4.2\text{ K},\varepsilon)/B_{c2}(4.2\text{ K},0)]^{1/w}$ ($w\approx 3$) can be satisfied only in a certain case. From the ratio of the thermodynamic critical field and the upper critical field, the Ginzburg-Landau parameter κ for Nb_3Sn is calculated which is substantially different from the Summers refinement. The strain and temperature dependences of the superconducting parameters for Nb_3Sn strands are summarized in Sec. IV, and the Kramer flux-line lattice (FLL) shearing model is compared with the reported experimental data.

II. THE TRANSITION TEMPERATURES AND THE THERMODYNAMIC CRITICAL FIELDS

The transition temperature and the thermodynamic critical field can be calculated from the following Eliashberg equations:[15,21]

$$\tilde{\Delta}(n)=\pi T\sum_m[\lambda(\omega_n-\omega_m)-\mu^*]\frac{\tilde{\Delta}(m)}{[\tilde{\omega}^2(m)+\tilde{\Delta}^2(m)]^{1/2}},$$

$$\tilde{\omega}(n)=\omega_n+\pi T\sum_m\lambda(\omega_n-\omega_m)\frac{\tilde{\omega}(m)}{[\tilde{\omega}^2(m)+\tilde{\Delta}^2(m)]^{1/2}}, \qquad (1)$$

where $\tilde{\Delta}(n)$ and $\tilde{\omega}(n)$ are the renormalized energy gap and the Matubara frequency $[\omega_n=\pi T(2n+1)]$, respectively. μ^* is the Coulomb pseudopotential, and $\lambda(\omega_n-\omega_m)$ is defined as follows:

$$\lambda(\omega_n)=2\int_0^\infty d\omega\frac{\omega}{\omega^2+\omega_n^2}\alpha^2(\omega)F(\omega). \qquad (2)$$

Near the transition temperature, the above nonlinear Eliashberg equations (1) are reduced to following linear equations:[17,19,20]

$$\tilde{\Delta}(n)=\pi T\sum_m[\lambda(\omega_n-\omega_m)-\mu^*]\frac{\tilde{\Delta}(m)}{|\tilde{\omega}(m)|}=\sum_m\tilde{M}_{nm}\tilde{\Delta}(m),$$

$$\tilde{\omega}(n)=\omega_n+\pi T\sum_m\lambda(\omega_n-\omega_m)\operatorname{sgn}(\tilde{\omega}_m). \qquad (3)$$

Solving the matrix equation $\det|\tilde{M}-\tilde{I}|=0$ gives the transition temperature.

Here we make an approximation for $\lambda(\omega_n-\omega_m)$ as

$$\lambda(\omega_n)=2\int_0^\infty d\omega\frac{\omega}{\omega^2+\omega_n^2}\alpha^2(\omega)F(\omega)\approx\frac{\lambda\langle\omega^2\rangle}{\langle\omega^2\rangle+\omega_n^2}, \qquad (4)$$

where the coupling constant λ and the charateristic phonon frequency $\langle\omega^2\rangle^{1/2}$ are defined as

$$\lambda=2\int_0^\infty d\omega\frac{1}{\omega}\alpha^2(\omega)F(\omega),$$

$$\langle\omega^2\rangle=\frac{2}{\lambda}\int_0^\infty d\omega\omega\alpha^2(\omega)F(\omega). \qquad (5)$$

It is equivalent to introducing an Einstein spectrum for the electron-phonon spectrum density function $\alpha^2(\omega)F(\omega)$ described by the above two parameters,

$$\alpha^2(\omega)F(\omega)=\lambda\langle\omega^2\rangle^{1/2}\delta(\omega-\langle\omega^2\rangle^{1/2}). \qquad (6)$$

This approximation has been used for the transition temperature calculation by Kresin and co-workers[19,20] and for the upper critical field by Bulaevski *et al.*[22] It is known to be appropriate for intermediate coupling $(\lambda=1-2)$.[18,23–25]

The electron-phonon spectrum density $\alpha^2(\omega)F(\omega)$ was determined from tunneling measurements[26–30] but the strain dependence of $\alpha^2(\omega)F(\omega)$ is not yet known. The relationship between λ and $\langle\omega^2\rangle^{1/2}$, $\lambda=N(0)\langle I^2\rangle/M\langle\omega^2\rangle=\eta/M\langle\omega^2\rangle$, was found by McMillan, where $N(0)$ is the density of states for quasiparticle excitations at the Fermi surface, $\langle I^2\rangle$ is the average over the Fermi surface of the electron-phonon matrix element, M is the atomic mass, and η is the McMillan-Hopfield parameter.[31,32] It was reported that the critical temperature is strongly affected by the variation in η.[17] The hydrostatic pressure dependences of λ, $N(0)$, and $\langle I^2\rangle$ were estimated from the heat-capacity coefficient γ by Lim *et al.*[33] In their work, a linear pressure dependence of $\langle\omega^2\rangle^{1/2}$ was assumed. The strain dependence of $N(0)$ was considered as a possible explanation for the strain effect of A15 superconductors.[34] It was disputed by the strain effects of ternary Nb_3Sn strands.[35,36] The addition of third element broadens the peak in the density of state and it is expected to reduce the strain effects. However, ternary strands are usually more strain sensitive than binary strands. Another proposed mechanism for the strain effects is phonon anharmonicity.[37] Recently, Markiewicz estimated the anharmonic strain function from the strain dependence of $T_c(\varepsilon)$ and calculated the strain dependences of λ and $\langle\omega^2\rangle^{1/2}$. In his model, the strain dependence of $T_c(\varepsilon)$ is largely due to the strain dependence of λ.[14]

In this work, we do not rely on any specific model for the strain dependence of $T_c(\varepsilon)$. Instead, we estimate the strain dependences of $\lambda(\varepsilon)$ and $\langle\omega^2\rangle^{1/2}(\varepsilon)$ from the empirical strain dependence of $T_c(\varepsilon)$ to calculate the strain and temperature dependences of $B_c(T,\varepsilon)$ and $B_{c2}(T,\varepsilon)$. General conditions needed to satisfy the empirical relation $T_c(\varepsilon)/T_c(0)=[B_{c2}(4.2\text{ K},\varepsilon)/B_{c2}(4.2\text{ K},0)]^{1/w}$ ($w\approx 3$) are studied. Since T_c is approxiamtely proportional to $\lambda\langle\omega^2\rangle^{1/2}$ for intermediate coupling,[17] the strain-induced T_c decrease is either due to the reduction of λ or $\langle\omega^2\rangle^{1/2}$ or both. The following three representative cases are considered. (i) Case I: $\langle\omega^2\rangle^{1/2}$ remains constant (T_c is a function of λ only), (ii) case II: λ is constant, and (iii) case III: λ and $\langle\omega^2\rangle^{1/2}$ are proportional to each other $(\lambda/\lambda_{max}=\langle\omega^2\rangle^{1/2}/\langle\omega^2\rangle^{1/2}_{max})$. The importance of the low-energy-phonon softening and its relevance for the reduction of T_c were argued by Testardi.[37] Case I is an extreme representative case for the phonon anharmonicity model. The application of strain mostly affects the low-frequency mode of the electron-phonon spectrum density function $\alpha^2(\omega)F(\omega)$ and as a result only the coupling constant λ varies. In the

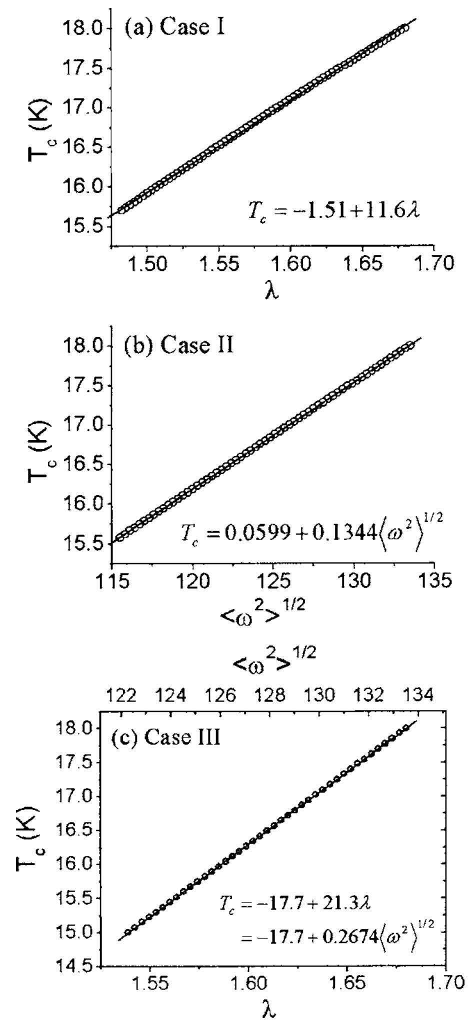

FIG. 1. Calculated critical temperature as a function of λ or $\langle\omega^2\rangle^{1/2}$ for each case [(c): λ dependence, bottom x axis; $\langle\omega^2\rangle^{1/2}$ dependence, top x axis]. Solid lines are linear fits to the data.

electronic models,[38] such as the Labbé-Friedel model, the phonon softening is assumed for the very long wavelength only, which corresponds to case II. Case III is in between cases I and II, both low- and high-frequency mode softenings of electron-phonon spectrum density function $\alpha^2(\omega)F(\omega)$ are important. A linear relation between λ and $\langle\omega^2\rangle^{1/2}$ is considered as an example.

For the calculation of the transition temperature, $\mu^* = 0.157$ and $\lambda(\varepsilon=0)=1.68$ are used which were obtained from tunneling measurements.[17,18] $\langle\omega^2\rangle^{1/2}(\varepsilon=0)$ of 133.5 K, which is comparable to the reported values,[17,18] is a calculated value to get $T_c(\varepsilon=0)=18$ K. The transition temperatures $T_c(\varepsilon=0)$ in the range of 17.0–18.5 K were reported for Nb_3Sn strands.[2–5,13] Figure 1 presents the transition temperature as a function of λ or $\langle\omega^2\rangle^{1/2}$ for each case. Consistent with the result of Allen and Dynes[17] and Kresin,[20] T_c decreases almost linearly with the decrease of λ or $\langle\omega^2\rangle^{1/2}$. Empirically, the strain dependence of $T_c(\varepsilon)$ can be written as $T_c(\varepsilon)=T_c(0)(1-\alpha|\varepsilon|^{1.7})$, where $\alpha \approx 300$ for binary strands under compressive strain.[2–4,9] The transition temperatures within the temperature range from $\sim 15.85[=T_c(\varepsilon=-1\%)]$ to 18 K are fitted linearly. The strain dependences of $\lambda(\varepsilon)$ and $\langle\omega^2\rangle^{1/2}(\varepsilon)$ for each case are listed in Table I. Without loss of generality, we consider only the compressive strain dependence with $\alpha=300$.

In order to calculate the thermodynamic critical field we need to solve the nonlinear Eliashberg equations [Eqs. (1)]. The thermodynamic critical field is defined as $B_c(T) = [8\pi(F_n - F_s)]^{1/2}$, and the free-energy difference between the normal and the superconducting states can be calculated from the renormalized gap $\tilde{\Delta}(n)$ and the Matsubara frequency $\tilde{\omega}(n)$.[21]

$$\frac{F_n - F_s}{N(0)} = 2\pi T \sum_n \omega_n \left\{ \frac{\tilde{\omega}(n)}{[\tilde{\omega}^2(n) + \Delta^2(n)]^{1/2}} - \operatorname{sgn}\omega_n \right\} + \pi^2 T^2 \sum_{n,m} \left(\left\{ \frac{\tilde{\omega}(n)}{[\tilde{\omega}^2(n) + \tilde{\Delta}^2(n)]^{1/2}} \times \frac{\tilde{\omega}(m)}{[\tilde{\omega}^2(m) + \tilde{\Delta}^2(m)]^{1/2}} - \operatorname{sgn}(\omega_n\omega_m) \right\} \times \lambda(\omega_n - \omega_m) + \frac{\tilde{\Delta}(n)}{[\tilde{\omega}^2(n) + \tilde{\Delta}^2(n)]^{1/2}} \frac{\tilde{\Delta}(m)}{[\tilde{\omega}^2(m) + \tilde{\Delta}^2(m)]^{1/2}} \times [\lambda(\omega_n - \omega_m) - \mu^*] \right). \quad (7)$$

The reported value of $B_c(T=0)$ at $\varepsilon=0$ is ~ 0.526 T, and the value of $N(0)$ used for the calculation is 105.2 states/Ry (unit cell).[39] It is smaller than the augumented plane-wave (APW) band calculation result[40] but close to the empirical estimation from the electronic heat-capacity coefficient γ.[41] The thermodynamic critical fields at zero temperature $B_c(T=0)$ as a function of λ or $\langle\omega^2\rangle^{1/2}$ are shown in Fig. 2. $B_c(0)$ can be described by a linear function of λ or $\langle\omega^2\rangle^{1/2}$ as summarized in Table II. We also present the strain dependence of $B_c(T=0,\varepsilon)$ obtained from the strain dependences of $\lambda(\varepsilon)$ and $\langle\omega^2\rangle^{1/2}(\varepsilon)$. The strain dependence of $B_c(T=0,\varepsilon)$ can be written in the same form of $T_c(\varepsilon)$ as $B_c(\varepsilon)=B_c(0)(1-\beta|\varepsilon|^{1.7})$. The temperature dependences of the thermodynamic critical field $B_c(T,\varepsilon=\text{const})$ are shown in Fig. 3. The values of λ and $\langle\omega^2\rangle^{1/2}$ are arbitrarily selected for each case within the range of the critical temperatures from ~ 15.8 to 18 K. It is found that the temperature dependence of the thermodynamic critical field can be written as $B_c(T,\varepsilon)=B_c(0,\varepsilon)(1-t^{2.17})$, where $t=T/T_c(\varepsilon)$ (solid lines in Fig. 3).

TABLE I. The strain dependences of $\lambda(\varepsilon)$ and $\langle\omega^2\rangle^{1/2}(\varepsilon)$ obtained the empirical transition temperature relation $T_c(\varepsilon)=T_c(0)(1-\alpha|\varepsilon|^{1.7})$, with $\alpha=300$. $\lambda(0)$ and $\langle\omega^2\rangle^{1/2}(0)$ are 1.68 and 133.5 K, respectively.

	$\lambda(\varepsilon)$	$\langle\omega^2\rangle^{1/2}(\varepsilon)$
Case I	$\lambda(\varepsilon)=\lambda(0)(1-277\|\varepsilon\|^{1.7})$	$\langle\omega^2\rangle^{1/2}(\varepsilon)=\langle\omega^2\rangle^{1/2}(0)$
Case II	$\lambda(\varepsilon)=\lambda(0)$	$\langle\omega^2\rangle^{1/2}(\varepsilon)=\langle\omega^2\rangle^{1/2}(0)(1-301\|\varepsilon\|^{1.7})$
Case III	$\lambda(\varepsilon)=\lambda(0)(1-151\|\varepsilon\|^{1.7})$	$\langle\omega^2\rangle^{1/2}(\varepsilon)=\langle\omega^2\rangle^{1/2}(0)(1-151\|\varepsilon\|^{1.7})$

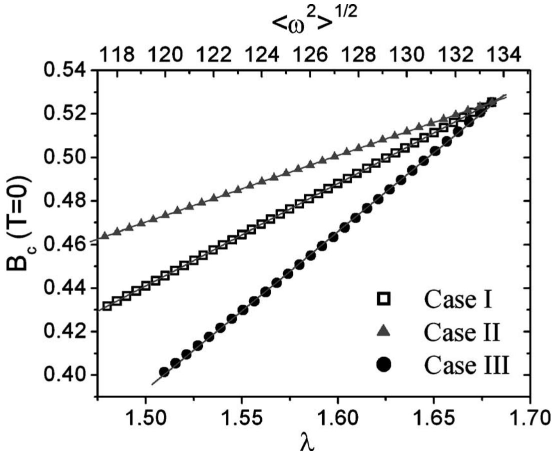

FIG. 2. Calculated thermodynamic critical field as a function of λ or $\langle\omega^2\rangle^{1/2}$ for each case (case I: bottom x axis, case II: top x axis, and case III: λ dependence, bottom x axis; $\langle\omega^2\rangle^{1/2}$ dependence, top x axis). Solid lines are linear fits to the data. Calculations were done at 0.5 K. $[B_c(T=0,\varepsilon) \cong B_c(0.5\text{ K},\varepsilon)$, see Fig. 3.]

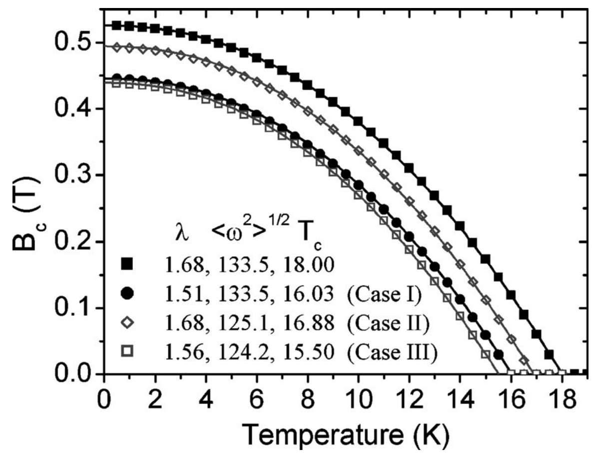

FIG. 3. The temperature dependence of the thermodynamic critical field for representative values of λ and $\langle\omega^2\rangle^{1/2}$. All lines are calculated with $B_c(T,\varepsilon)=B_c(0,\varepsilon)(1-t^{2.17})$, where $t=T/T_c(\varepsilon)$.

III. THE UPPER CRITICAL FIELD AND THE GINZBURG-LANDAU PARAMETER

A strong-coupling theory of the upper critical field was developed by Schossmann and Schachinger,[42] which is an extension of the Werthamer-Helfand-Hohenberg theory.[43] For the nonmagnetic impurity concentration specified by $t_+ = 1/(2\pi\tau)$ with the scattering time τ, the equations are

$$\tilde{\Delta}(n) = \pi T \sum_m \frac{[\lambda(\omega_n - \omega_m) - \mu^*]\tilde{\Delta}(m)}{\chi^{-1}(\tilde{\omega}_m) - \pi t_+},$$

$$\tilde{\omega}(n) = \omega_n + \pi T \sum_m \lambda(\omega_n - \omega_m)\text{sgn}(\tilde{\omega}_m) + \pi t_+ \,\text{sgn}(\tilde{\omega}_n),$$

with

$$\chi(\tilde{\omega}_n) = \frac{2}{\sqrt{\alpha}} \int_0^\infty dq \exp(-q^2)\tan^{-1}\left(\frac{q\sqrt{\alpha}}{|\tilde{\omega}|}\right). \quad (8)$$

The eigenvalue α gives the upper critical field according to $\alpha = \pi B_{c2} v_F{}^2 / 2\phi_0$, where ϕ_0 is the quantum flux, and v_F is the electron Fermi velocity. For Nb_3Sn superconductors, a wide range of the upper critical field around 28 T (26–30 T) was reported.[44] The electron Fermi velocity v_F was fitted for each impurity concentration to calculate the upper critical field $B_{c2}(T=0)$ of 28 T at $\varepsilon=0$. For the clean limit, $\pi t_+ = 1$ K, the fitted electron Fermi velocity v_F is 0.1548×10^6 m/s and for the dirty limit, $\pi t_+ = 1000$ K, it is 0.3895×10^6 m/s. These values are comparable to the APW band calculation result of 0.284×10^6 m/s.[45] The upper critical fields $B_{c2}(0)$ for various impurity concentrations are presented as a function of λ or $\langle\omega^2\rangle^{1/2}$ in Fig. 4. For all model cases studied in this work, $B_{c2}(0)$ decreases more rapidly with the decrease of λ or $\langle\omega^2\rangle^{1/2}$ in the clean limit than in the dirty limit. Bulaevskii *et al.* reported similar behavior for $\lambda \gg 1$. They showed that $B_{c2}(0)$ is propotional to $\lambda^2\langle\omega^2\rangle$ in the clean limit, whereas $B_{c2}(0) \propto \lambda\langle\omega^2\rangle^{1/2}$ in the dirty limit.[22]

TABLE II. λ or $\langle\omega^2\rangle^{1/2}$ dependences of $B_c(T=0)$ and the strain dependences of $B_c(T=0,\varepsilon)$ for each case.

	λ or $\langle\omega^2\rangle^{1/2}$ dependence	Strain dependence
Case I	$B_c(0)=-0.2618+0.4696\lambda$	$B_c(0,\varepsilon)=B_c(0,0)(1-412\lvert\varepsilon\rvert^{1.7})$
Case II	$B_c(0)=0.01106+0.00385\langle\omega^2\rangle^{1/2}$	$B_c(0,\varepsilon)=B_c(0,0)(1-295\lvert\varepsilon\rvert^{1.7})$
Case III	$B_c(0)=-0.7288+0.7462\lambda$	$B_c(0,\varepsilon)=B_c(0,0)(1-360\lvert\varepsilon\rvert^{1.7})$
	$B_c(0)=-0.7288+0.00939\langle\omega^2\rangle^{1/2}$	

$B_{c2}(0)$ is not a linear function of λ or $\langle\omega^2\rangle^{1/2}$ as shown in Fig. 4. However, the data within the upper critical field ranging from ~24.9 to 28 T can be fitted linearly. The solid lines in Fig. 4 are the fitting results for the clean limit ($\pi t_+ = 1$ K). Within this range, the strain dependence of $B_{c2}(T=0,\varepsilon)$ can be describe by $B_{c2}(0,\varepsilon)=B_{c2}(0,0)(1-a|\varepsilon|^{1.7})$. The empirical relation $T_c(\varepsilon)/T_c(0)=[B_{c2}(4.2\text{ K},\varepsilon)/B_{c2}(4.2\text{ K},0)]^{1/w}$ ($w \approx 3$) can be rewritten as a relation between the coefficients of $T_c(\varepsilon)$ and $B_{c2}(T=0,\varepsilon)$ as $a \approx 3\alpha$. (In the inset of Fig. 6, the strain dependences of $B_{c2}(T,\varepsilon)/B_{c2}(T,0)$ at 0 and 4.2 K are compared. The difference in the value of coefficient a is less than 5%.) For compressive strain, $a \approx 900$, and the lower bound of ~24.9 T for the above linear fitting corresponds to $B_{c2}(T=0,\varepsilon)$ at −0.5% strain. This linearization is valid up to 0.8% compressive strain within 2% error but the deviation increases to 7% at −1% strain. (The dashed horizontal lines in Fig. 4 correspond to $B_{c2}(0,\varepsilon=-0.8\%) \approx 21.1$ T.) The coefficients a for various impurity concentration levels obtained from the strain dependences of $\lambda(\varepsilon)$ and $\langle\omega^2\rangle^{1/2}(\varepsilon)$ are listed in Table III. The $a \approx 3\alpha$ relation is satisfied only in case I at the low impurity concentration level. Our calculation results seem to support the phonon anharmonicity model. The empirical relation between $T_c(\varepsilon)$ and $B_{c2}(4.2\text{ K},\varepsilon)$ is mainly due to the low-energy-phonon softening. Recently, for ternary strands, w of ~2.5 or even below 2 were reported.[46] Our results suggest that it is either due to the increase of the impurity concentration or due to the change of the strain dependence of $\alpha^2(\omega)F(\omega)$, from case I to case III, by the addition of ternary element.

The Ginzburg-Landau parameter κ is given by the ratio of the thermodynamic and the upper critical field. It is now clear that the strain dependence of $\kappa(T=0,\varepsilon)$ at zero tem-

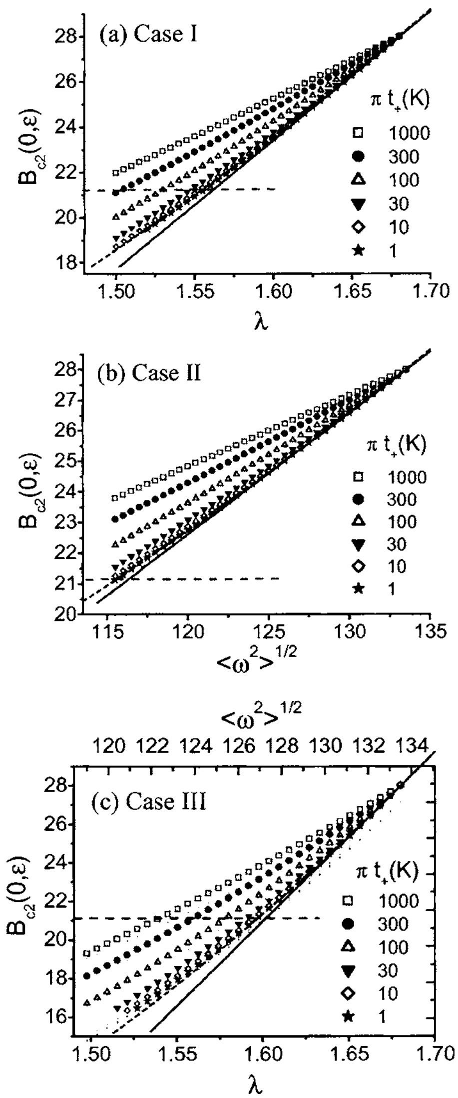

FIG. 4. Calculated upper critical field as a function of λ or $\langle\omega^2\rangle^{1/2}$ for each case [(c): λ dependence, bottom x axis; $\langle\omega^2\rangle^{1/2}$ dependence, top x axis]. Calculations were done at 0.5 K. [$B_{c2}(T=0,\varepsilon)\cong B_{c2}(0.5\text{ K},\varepsilon)$, see Fig. 6.] Solid lines are linear fits to the clean limit ($\pi t_+=1$ K) data within the upper critical field range from 24.9 to 28 T. Dotted lines are calculated with Eq. (9) using the strain dependences of $\lambda(\varepsilon)$ and $\langle\omega^2\rangle^{1/2}(\varepsilon)$ listed in Table I. Dashed horizontal lines are guidelines which correspond to $B_{c2}(0,\varepsilon=-0.8\%)=21.1$ T.

perature can be written as $\kappa(0,\varepsilon)=\kappa(0,0)(1-\gamma|\varepsilon|^{1.7})$. The values of γ are 522 and 480, for example, for case I, and 423 and 383 for case III with the impurity concentrations of $\pi t_+=10$ and 30 K, respectively. We rewrite the strain dependence of $B_{c2}(T=0,\varepsilon)$ as

TABLE III. The coefficients a for the strain dependence of $B_{c2}(T=0,\varepsilon)$ [$=B_{c2}(0,0)(1-a|\varepsilon|^{1.7})$] at various impurity concentrations for each case.

	πt_+				
a	1 K	10 K	30 K	100 K	1000 K
Case I	924	904	866	770	567
Case II	566	555	529	469	338
Case III	790	773	740	656	476

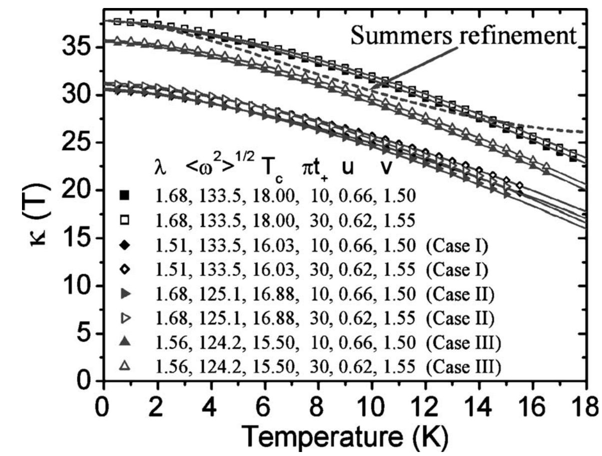

FIG. 5. The temperature dependence of the Ginzburg-Landau parameter κ calculated for representative values of λ and $\langle\omega^2\rangle^{1/2}$. Dotted line represents the Summers expression for the temperature dependence of κ. Solid lines are calculated with Eq. (10). The fitting parameters u and v are listed in the figure.

$$B_{c2}(0,\varepsilon)=\sqrt{2}B_c(0,\varepsilon)\cdot\kappa(0,\varepsilon) = B_{c2}(0,0)(1-\beta|\varepsilon|^{1.7})(1-\gamma|\varepsilon|^{1.7}), \tag{9}$$

where β and γ are the coefficients for the strain dependences of $B_c(0,\varepsilon)$ and $\kappa(0,\varepsilon)$. It gives better fits to the calculated data as can be seen in Fig. 4. The dotted lines in Fig. 4 are calculated with Eq. (9) using the strain dependences of $\lambda(\varepsilon)$ and $\langle\omega^2\rangle^{1/2}(\varepsilon)$ listed in Table I. The higher-order polynomial fits for the strain dependences of superconducting parameters were also proposed by Hampshire and co-workers.[5,46] $T_c(\varepsilon)$ and $B_{c2}(0,\varepsilon)$ were fitted with fourth-order polynomials in their interpolative scaling law.

The temperature dependences of $\kappa(T)$ calculated for various values of λ and $\langle\omega^2\rangle^{1/2}$ are presented in Fig. 5. The Summers expression $\kappa(T)=\kappa(0)[1-0.31t^2(1-1.77\ln t)]$ deviates from our calculation results (dotted line).[13] For low impurity concentrations ($\pi t_+\leqslant 100$ K), it is found that the Ginzburg-Landau parameter $\kappa(T,\varepsilon)$ can be fitted to the following equation.

$$\kappa(T,\varepsilon)=\frac{\kappa(0,\varepsilon)}{1+u\{[\kappa(0,\varepsilon)/\kappa(0,0)][T_c(0)/T_c(\varepsilon)]\}} \times\left[1+u\frac{\kappa(0,\varepsilon)}{\kappa(0,0)}\frac{T_c(0)}{T_c(\varepsilon)}(1-t^v)\right], \tag{10}$$

where u and v are the fitting parameters. The strain and temperature dependences of $\kappa(T,\varepsilon)$ are not a separable function, and the temperature dependence of $\kappa(T,\varepsilon)$ is affected by the strain dependence of $T_c(\varepsilon)$. All solid lines in Fig. 5 are calculated with Eq. (10). The fitting parameters u and v depend only on the impurity concentration, as listed in Fig. 5. The overall strain and temperature dependences of the upper critical fields can be written as

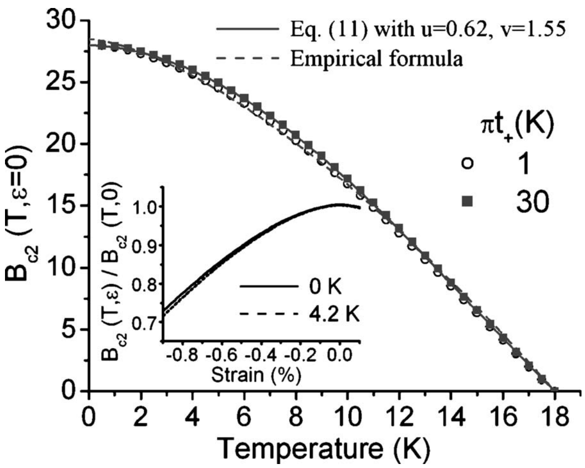

FIG. 6. The temperature dependence of $B_{c2}(T,\varepsilon=0)$ for the impurity concentration of $\pi t_+=30$ and 1 K. Solid line is calculated with Eq. (11) using $u=0.62$ and $v=1.55$. Dotted line represents the empirical fitting formula $B_{c2}(T,\varepsilon)=B_{c2}(0,\varepsilon)(1-t^v)$, with $\nu=1.5$. Inset: comparison of the strain dependence of $B_{c2}(T,\varepsilon)$ at 0 and 4.2 K. $B_{c2}(T,\varepsilon)$ are calculated with Eq. (11) for case I with the impurity concentration of $\pi t_+=30$ K.

$$B_{c2}(T,\varepsilon) = \sqrt{2}B_c(0,\varepsilon)(1-t^{2.17}) \times\left(\frac{\kappa(0,\varepsilon)}{1+u\{[\kappa(0,\varepsilon)/\kappa(0,0)][T_c(0)/T_c(\varepsilon)]\}}\right) \times\left\{1+u\frac{\kappa(0,\varepsilon)}{\kappa(0,0)}\frac{T_c(0)}{T_c(\varepsilon)}(1-t^v)\right\}. \quad (11)$$

Typical temperature dependences of the upper critical field at $\varepsilon=0$ calculated from the Schossmann and Schachinger theory are shown in Fig. 6. A solid line in Fig. 6 is calculated with Eq. (11) using the same fitting parameters u and v for $\pi t_+=30$ K in Fig. 5. The strain dependences of $B_{c2}(T,\varepsilon)$ calculated with Eq. (11) at 0 and 4.2 K for case I with $\pi t_+=30$ K are shown in the inset of Fig. 6. It was reported that $B_{c2}(T,\varepsilon)$ of Nb_3Sn can be fitted to $B_{c2}(T,\varepsilon)=B_{c2}(0,\varepsilon)(1-t^v)$ with $\nu=1.5$.[44] Our calculation results from Eq. (8) are compatible to this empirical fitting formula, which is shown as a dotted line in Fig. 6. The strain and temperature dependences of $B_{c2}(T,\varepsilon)$ derived from the microscopic calculations are in a good agreement with emprical observations on inhomogeneous Nb_3Sn samples including commercial wires. It should be noted that the inhomogeneity in Nb_3Sn strands was not explicitly considered in our calculation. However, it is implicitly included through the assumption of the strain dependence of $T_c(\varepsilon)$ in the form of Ekin's empirical relation.

IV. A SCALING LAW FOR THE CRITICAL CURRENT

If we summarize the above results, the strain and temperature dependences of the superconducting parameters can be written as

$$T_c(\varepsilon) = T_c(0)(1-\alpha|\varepsilon|^{1.7}), \quad (12)$$

$$B_c(T,\varepsilon) = B_c(0,0)(1-\beta|\varepsilon|^{1.7})(1-t^{2.17}), \quad (13)$$

$$\kappa(T,\varepsilon) = \frac{\kappa(0,0)(1-\gamma|\varepsilon|^{1.7})}{1+u[(1-\gamma|\varepsilon|^{1.7})/(1-\alpha|\varepsilon|^{1.7})]} \times\left[1+u\frac{1-\gamma|\varepsilon|^{1.7}}{1-\alpha|\varepsilon|^{1.7}}(1-t^v)\right], \quad (14)$$

$$B_{c2}(T,\varepsilon) = \frac{B_{c2}(0,0)(1-\beta|\varepsilon|^{1.7})(1-\gamma|\varepsilon|^{1.7})}{1+u[(1-\gamma|\varepsilon|^{1.7})/(1-\alpha|\varepsilon|^{1.7})]}(1-t^{2.17}) \times\left[1+u\frac{1-\gamma|\varepsilon|^{1.7}}{1-\alpha|\varepsilon|^{1.7}}(1-t^v)\right], \quad (15)$$

where the coefficients α, β, and γ are for the strain depen-dece of $T_c(\varepsilon)$, $B_c(0,\varepsilon)$, and $\kappa(0,\varepsilon)$, respectively. Microscopically, α and β depend only on the strain dependence of $\alpha^2(\omega)F(\omega)$, while γ is also related to the level of the impurity concentration. The coefficient u and the exponent v describe the temperature dependence of $\kappa(T,\varepsilon)$ and depend on the impurity concentration. We further derive a scaling formula for the critical current following the Kramer FLL shearing model:[47]

$$F_p = I_c\times B = F_m(T,\varepsilon)[b^{1/2}(1-b)^2] \propto B_{c2}^{5/2}\kappa^{-2}[b^{1/2}(1-b)^2],$$

$$F_m(T,\varepsilon) = C\frac{(1-\beta|\varepsilon|^{1.7})^{5/2}(1-\gamma|\varepsilon|^{1.7})^{1/2}}{\{1+u[(1-\gamma|\varepsilon|^{1.7})/(1-\alpha|\varepsilon|^{1.7})]\}^{1/2}} \times(1-t^{2.17})^{5/2}\left\{1+u\frac{1-\gamma|\varepsilon|^{1.7}}{1-\alpha|\varepsilon|^{1.7}}(1-t^v)\right\}^{1/2}, \quad (16)$$

where, C is a coefficient independent of field, temperature, and strain. The critical current can be written as

$$I_c(B,T,\varepsilon) = \frac{C}{B_{c2}(0,0)}\frac{(1-\beta|\varepsilon|^{1.7})^{3/2}(1-\gamma|\varepsilon|^{1.7})^{-1/2}}{\{1+u[(1-\gamma|\varepsilon|^{1.7})/(1-\alpha|\varepsilon|^{1.7})]\}^{-1/2}} \times\frac{(1-t^{2.17})^{3/2}}{\{1+u[(1-\gamma\varepsilon^{1.7})/(1-\alpha\varepsilon^{1.7})](1-t^v)\}^{1/2}} \times[b^{-1/2}(1-b)^2]. \quad (17)$$

In the Kramer model, the pinning force is determined by the competition between single-vortex pinning and plastic FLL shearing around strong planar pins.[47] It was shown that the FLL shearing dynamic pinning force is proportional to its shear modulus C_{66}. But the assumption of the plastic FLL shearing motion was questioned[48] and recent statistical theories of weak pinning suggest that the FLL shearing pinning force is proportional to $1/C_{66}$.[49] Moreover, in his calculation the shear modulus C_{66} $[\propto B_{c2}{}^2\kappa^{-2}(1-b)^2]$ calculated by Labusch was used, which is valid near the upper critical field.[50] However, the Kramer-like relation $F_p=I_c\times B\propto b^{1/2}(1-b)^2$ is widely reported in the literature.[2–6,13,46,51] The Summers scaling law can be written in a form consistent with the Kramer model.[52] It was even reported that the pinning force within Nb_3Sn filaments can be described in the form of the Kramer model.[5]

We compare the above scaling formula based on the Kramer model and the strong-coupling theory with empirical

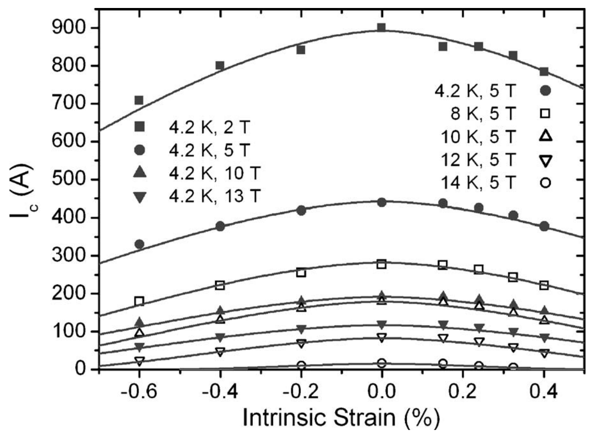

FIG. 7. Typical critical current measurement data at various fields, temperatures, and strains obtained by Godeke *et al.* All lines are calculated with Eq. (17) using the same parameters obtained from the fitting of $B_{c2}(T,\varepsilon)$ with Eq. (15).

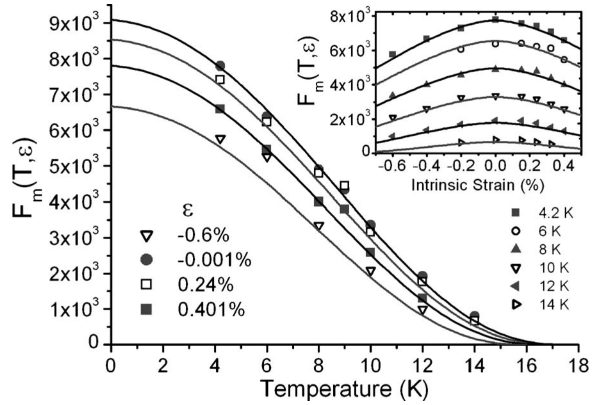

FIG. 9. The pinning force maximum obtained from the Kramer plots as a function of temperature at various strains. (Inset: the strain dependence of the pinning force maximum.) All lines are calculated with Eq. (16) using the same parameters obtained from the fitting of $B_{c2}(T,\varepsilon)$.

data reported in the literature. Typical critical current data obtained by Godeke *et al.*[4] for a Furukawa strand at various fields, temperatures, and strains are presented in Fig. 7. The upper critical field $B_{c2}(T,\varepsilon)$ and the pinning force maximum $F_m(T,\varepsilon)$ defined in Eq. (16) are extrapolated from the Kramer plots as shown in Figs. 8 and 9. At 4.2 K, $B_{c2}(T,\varepsilon)$ and $F_m(T,\varepsilon)$ can be approximated as

$$B_{c2}(4.2\text{ K},\varepsilon) \approx B_{c2}(4.2\text{ K},0)(1-\beta|\varepsilon|^{1.7})(1-\gamma|\varepsilon|^{1.7}),$$

$$F_m(4.2\text{ K},\varepsilon) \approx C(1-\beta|\varepsilon|^{1.7})^{5/2}(1-\gamma|\varepsilon|^{1.7})^{1/2}. \quad (18)$$

An estimation of β and γ can be made from the strain dependences of $B_{c2}(T,\varepsilon)$ and $F_m(T,\varepsilon)$ at 4.2 K. All parameters except C were determined from the fitting of $B_{c2}(T,\varepsilon)$. The best fit was obtained with $T_c(0)=17.5$ K, $B_{c2}(0,0)=32.0$ T, $\alpha=500$, $\beta=580$, $\gamma=630$, $u=0.85$, and $v=1.2$. The fitting results are shown in Fig. 8. All lines in Fig. 8 are calculated with Eq. (15) using the above parameters. The same fitting parameters give reasonable fits for $F_m(T,\varepsilon)$ and $I_c(B,T,\varepsilon)$ as can be seen in Figs. 7 and 9. Our results do not necessarily mean the physical validity of the Kramer model. The upper critical fields $B_{c2}(T,\varepsilon)$ shown in Fig. 8 are not the actual upper critical fields. The inconsistency between the extrapolated and actual critical fields was reported from direct measurements of B_{c2}.[53] The Kramer extrapolation overestimates the upper critical field at low temperature and underestimates B_{c2} near T_c. But it should be noted that the strain dependence of $T_c(\varepsilon)$ used for the calculation is obtained from the empirical relation $T_c(\varepsilon)/T_c(0)=[B_{c2}(4.2\text{ K},\varepsilon)/B_{c2}(4.2\text{ K},0)]^{1/w}$. The empirical relation $B_{c2}(4.2\text{ K},\varepsilon)=B_{c2}(4.2\text{ K},0)(1-a|\varepsilon|^{1.7})$ is an expression for the strain dependence of the extrapolated upper critical field.[9]

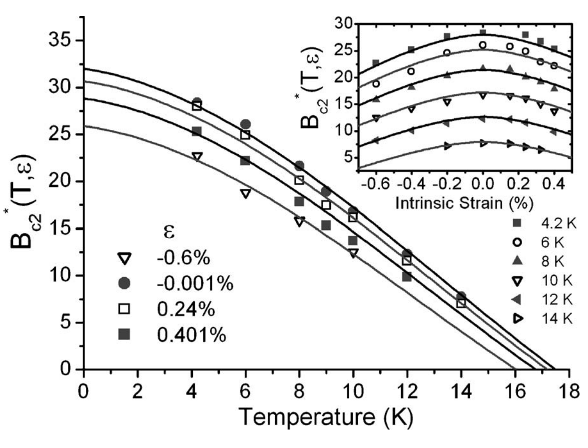

FIG. 8. The extrapolated upper critical field as a function of temperature at various strains. (Inset: the strain dependence of the extrapolated upper critical field.) The extrapolated upper critical fields are fitted with Eq. (15). Lines are the fitting results. The best fit is obtained with $T_c(0)=17.5$ K, $B_{c2}(0,0)=32.0$ T, $\alpha=500$, $\beta=580$, $\gamma=630$, $u=0.85$, and $v=1.2$.

From an engineering point of view, however, it is clear that the Kramer model gives a reasonable unified scaling law for the critical current of Nb_3Sn strands if proper expressions for $\kappa(T,\varepsilon)$ and $B_{c2}(T,\varepsilon)$ are used. One of the merit of the proposed scaling law is its simplicity. Only eight parameters including $T_c(0)$ and $B_{c2}(0,0)$ are used. Moreover, all parameters except C can be found from the fitting of $B_{c2}(T,\varepsilon)$ and no additional fitting procedure is needed for the critical current data. The other merit of the proposed scaling formula is its relation with the microscopic theory. Since the coefficients β and γ are obtained from linear dependences of T_c, B_c, and κ as a function of λ or $\langle\omega^2\rangle^{1/2}$ in the previous sections, the increase of α implies the proportional increases of β and γ. To compare the above fitting parameters $\alpha=500$, $\beta=580$, and $\gamma=630$ with the calculation results, the calculated coefficients β and γ in the previous sections should be scaled by a factor of 5/3. ($a=1200-1400$ are reported for ternary Nb_3Sn strands under compressive strain,[54] which could be expressed as $\alpha=400-500$. For $\alpha=500$, the lower bound for the liner fitting, 15.85 K, corresponds to $\varepsilon\approx-0.75\%$.) Instead, we scale down the above fitting results for easy comparison, and the scaled fitting parameters are $\alpha_{\text{scaled}}=300$, $\beta_{\text{scaled}}=348$, and $\gamma_{\text{scaled}}=378$. These are similar to the calculation results for case III with the impurity concentration of $\pi t_+=30$ K, where the calculated coefficients are $\beta=360$ and $\gamma=383$. For the Furukawa strand, it can be argued that the impurity concentration is relatively low, and the strain effects are due to the overall phonon mode softening both at low and high frequencies. (If we rewrite the above fitting results of $T_c(\varepsilon)$ and $B_{c2}(T,\varepsilon)$ in the form of the conventional empirical relation $T_c(\varepsilon)/T_c(0)=[B_{c2}(4.2\ \mathrm{K},\varepsilon)/B_{c2}(4.2\ \mathrm{K},0)]^{1/w}$, the value of w is ~2.3.) A comparative study on the variation of these parameters with various fabrication methods and a different amount of ternary addition will be helpful to understand the observed difference in the strain effects.

V. CONCLUSIONS

In summary, we numerically calculated the transition temperature, the thermodynamic, and the upper critical fields of Nb_3Sn from Eliashberg theory with the Einstein spectrum. Three model cases for the variation of the coupling constant λ and the characteristic frequency $\langle\omega^2\rangle^{1/2}$ are considered. The empirical relation $T_c(\varepsilon)/T_c(0)=[B_{c2}(4.2\ \mathrm{K},\varepsilon)/B_{c2}(4.2\ \mathrm{K},0)]^{1/w}$ ($w\approx 3$) is satisfied only for the case where the low-frequency phonon mode softening is dominant, which support the phonon anharmonicity model. The observed variation of w is attributed either to the increase of the impurity concentration or to the change in the strain dependence of $\alpha^2(\omega)F(\omega)$. Analytic expressions for the strain and temperature dependences of $B_c(T,\varepsilon)$, $B_{c2}(T,\varepsilon)$, and $\kappa(T,\varepsilon)$ are formulated. The strain and temperature dependences of $\kappa(T,\varepsilon)$ are not a separable function, whereas in the Summers expression only the temperature dependence is considered. The Summers refinement shows deviation from our calculation results.

We propose a unified scaling law based on the Kramer model and the strong-coupling theory. It is found that the Kramer FLL shearing model with the analytic expression on $\kappa(T,\varepsilon)$ and $B_{c2}(T,\varepsilon)$ derived in this work gives reasonable fits to the critical current measurements on a Furukawa strand. From the analysis of the fitting parameters, it is argued that the impurity concentration is relatively low, and the strain effects are due to the phonon mode softening both at high and low frequencies.

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